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Consistent modified gravity: dark energy, acceleration and the absence of cosmic doomsday

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Abstract: We discuss the modified gravity which includes negative and positive powers of the curvature and which provides the gravitational dark energy. It is shown that in GR plus the term containing negative power of the curvature the cosmic speed-up may be achieved, while the effective phantom phase (with w less than -1) follows when such term contains the fractional positive power of the curvature. The minimal coupling with matter makes the situation more interesting: even $1/R$ theory coupled with the usual ideal fluid may describe the (effective phantom) dark energy. The account of R^2 term (consistent modified gravity) may help to escape of cosmic doomsday.

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Contents

1. Introduction

The interpretation of the very recent observational data indicates that current universe is flat and is in accelerating expansion which started about five billion years ago. The important characteristic of the (accelerated) expansion is so-called "equation of state" parameter w (for dominant energy density contribution) which is the relation between the pressure and the energy density of the universe. It is estimated that about 70 percent of universe energy density is composed of some mysterious effective fluid (dark energy which rules the universe) with negative pressure and w being close to -1. Despite the number of efforts we are still far away from the theoretical understanding of dark energy and its origin.

The very promising approach to dark energy is related with the (phenomenological) modifications of Einstein gravity in such a way, that it would give the gravitational alternative to dark energy. Conceptually, it looks very attractive as then the presence of dark energy is only the consequence of the universe expansion. One such model containing $1/R$ term (which may originate from M-theory [2]) was proposed in ref.[1] as gravitational alternative for dark energy. Modified gravity describes the accelerated expansion but it contains number of instabilities [3]. Nevertheless, further modification of the model by R^2-term [4] or $\ln R$-term [5] (see also [6]) leads to consistent modified gravity which may pass solar system tests and is free of the instabilities.[1] One may consider other generalizations like ones including the positive (negative) powers of Ricci tensor squared [8], coupling of $f(R)$ theory with scalar [9, 10], multidimensional $1/R$ theory [11] or even more extravagant (non-symmetric)

[1]It is interesting that Palatini formulation of modified gravity leads to physically different theory (for very recent discussion and list of refs., see [7]) which seems to be free of (some) instabilities too.

gravity (for recent discussion, see [12]). Definitely, various predictions of consistent modified gravity should be tested. In its own turn, these tests may suggest further modifications giving true description of an observable universe.

The present work is devoted to further study of the properties of modified gravity which contains positive and negative powers of the curvature. It is demonstrated that (effective phantom) dark energy cosmological solutions for the model naturally contain the finite-time, sudden singularity in the future (section 2). However, the consistent modified gravity with negative (or positive, fractional) powers of the curvature as well as with R^2 term has more stable future history. The cosmic doomsday does not occur there, rather the universe ends up in deSitter phase. In the third section it is considered the coupling of modified gravity with the usual matter. The very interesting observation shows that in such unified framework it is easier to realize various types of effectively dark energy universe. In summary, some outlook is given.

2. Sudden future singularity in modified gravity

Let us start from the general model of gravity depending only on curvature:

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} f(R) \ . \tag{2.1}$$

Here $f(R)$ can be an arbitrary function. It has been known [13] that such modified gravity theories can be rewritten into scalar-tensor form via the conformal transformation. Depending on the form of the function f the scalar-tensor theory may contain ghost-like term (with negative kinetic energy). It is quite remarkable that $f(R)$ gravity of special form as we show below from this big class of theories may find the interesting applications as candidate to describe dark energy universe and its acceleration. In a sense, it is return of somehow forgotten generalized gravity motivated by recent astrophysical data.

By introducing the auxilliary fields A, B, one may rewrite the action (2.1) as $S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left\{ B \left(R - A \right) + f(A) \right\}$. Using equation of motion and expressing B in terms of A, one arrives at the Jordan frame action. Using the conformal transformation $g_{\mu\nu} \to \mathrm{e}^{\sigma} g_{\mu\nu}$ with $\sigma = -\ln f'(A)$ (see also[14]), we obtain the Einstein frame action[4]:

$$S_E = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left(R - \frac{3}{2} g^{\rho\sigma} \partial_\rho \sigma \partial_\sigma \sigma - V(\sigma) \right) \ , \tag{2.2}$$

$$V(\sigma) = \mathrm{e}^{\sigma} g\left(\mathrm{e}^{-\sigma}\right) - \mathrm{e}^{2\sigma} f\left(g\left(\mathrm{e}^{-\sigma}\right)\right) = \frac{A}{f'(A)} - \frac{f(A)}{f'(A)^2} \ . \tag{2.3}$$

Note that two such theories in these frames are mathematically equivalent. However, physics seems to be different. For instance, in Einstein frame the matter does not freely fall along the geodesics which is well-established fact.

As an interesting example the action of (large distances) modified gravity may be taken in the following form

$$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\left(R - \gamma R^{-n}\right) \ . \tag{2.4}$$

Here γ is (an extremely small) coupling constant and n is some number. Since the function $f(A)$ and the scalar field σ are

$$f(A) = A - \gamma A^{-n} \ , \quad \sigma = -\ln\left(1 + n\gamma A^{-n-1}\right) \tag{2.5}$$

the potential is given by

$$V = \frac{\gamma(n+1)A^{-n}}{\left(1+n\gamma A^{-n-1}\right)^2} = \gamma(n+1)\left(\frac{\mathrm{e}^{-\sigma}-1}{n\gamma}\right)^{\frac{n}{n+1}}\mathrm{e}^{2\sigma} \ . \tag{2.6}$$

When curvature ($\sim A$) is small and $n > -1$ and $n \neq 0$, the potential behaves as an exponential function $V \sim \frac{1}{n\gamma}\left(1+\frac{1}{n}\right)A^{n+2} \sim \left(1+\frac{1}{n}\right)(n\gamma)^{\frac{1}{n+1}}\mathrm{e}^{\frac{n+2}{n+1}\sigma}$. On the other side, when the curvature is large, it follows $V \sim \gamma(n+1)A^{-n} \sim \frac{n+1}{n}(n\gamma)^{\frac{1}{n+1}}(-\sigma)^{\frac{n}{n+1}}$. Since $V'(A) = -\frac{\gamma n(n+1)A^{-n-1}\left\{1-(n+2)\gamma A^{-n-1}\right\}}{(1+n\gamma A^{-n-1})^3}$, $V(A)$ has only one extremum and the extremum is given at $A = \{(n+2)\gamma\}^{\frac{1}{n+1}}$.

The FRW universe metric in the Einstein frame is chosen as $ds_E^2 = -dt_E^2 + a_E^2(t_E)\sum_{i=1}^3\left(dx^i\right)^2$. If the curvature is small, the solution of equation of motion is $a_E \sim t_E^{\frac{3(n+1)^2}{(n+2)^2}}$. The FRW universe metric in Jordan frame is $ds^2 = -dt^2 + a^2(t)\sum_{i=1}^3\left(dx^i\right)^2$, where the variables in the Einstein frame and in the physical Jordan frame are related with each other by $t = \int \mathrm{e}^{\frac{\sigma}{2}}dt_E$, $a = \mathrm{e}^{\frac{\sigma}{2}}a_E$. This gives $t \sim t_E^{\frac{1}{n+2}}$ and

$$a \sim t^{\frac{(n+1)(2n+1)}{n+2}} \ , \quad w = -\frac{6n^2+7n-1}{3(n+1)(2n+1)} \ . \tag{2.7}$$

The first important consequence of above Eq.(2.7) is that there is possibility of accelerated expansion for some choices of n (effective quintessence). In fact if $n > \frac{-1+\sqrt{3}}{2}$ or $-1 < n < -\frac{1}{2}$, we find $w < -\frac{1}{3}$ and $\frac{d^2a}{dt^2} > 0$.[2] In other words, modified gravity presents the gravitational alternative for dark energy with the possibility of cosmic speed-up. The corresponding analysis was given in detail in ref.[5].

When $-1 < n < -\frac{1}{2}$, it follows $w < -1$. If $w < -1$, the universe is shrinking in the expression of a (2.7). If we replace the direction of time by changing t by

[2] In the solution (2.7), the Hubble parameter $H \equiv \frac{1}{a}\frac{da}{dt}$ has the form of $H = \frac{h_0}{t}$, where h_0 is a constant of the unity order. There is an ambiguity how to choose t in H. One natural choice is to take t to be of the order of the age T of the present universe. Since $T \sim 1.37\times 10^{10}$ years $\sim \left(10^{-33}\ \mathrm{eV}\right)^{-1}$, we find $H \sim 10^{-33}$ eV, which is consistent with the observed value of H in the present universe: $H_{\mathrm{observed}} \sim 70\,\mathrm{km\,s^{-1}Mpc^{-1}} \sim 10^{-33}\,\mathrm{eV}$. Then even if the original Lagrangian theory does not contain small parameter of the order of the Hubble parameter, such a small scale is naturally induced from the age of the universe in the power law expansion (2.7).

$-t$, the universe is expanding but t should be considered to be negative so that the scale factor a should be real. Then there appears a singularity at $t=0$, where the scale factor a diverges as $a \sim (-t)^{\frac{2}{3(w+1)}}$. One may shift the origin of the time by further changing $-t$ with $t_s - t$. Hence, in the present universe, t should be less than t_s and it looks to appear the singularity at $t = t_s$: $a \sim (t_s - t)^{\frac{2}{3(w+1)}}$. The future singularity may be called sudden or Big Rip singularity. We should note, however, the expression of w in (2.7) could be correct only when curvature is small, as in the present universe. When t goes to t_s, the curvatures become large as $(t_s - t)^{-2}$ and Eq.(2.7) becomes invalid. As the curvature becomes large, the first Einstein-Hilbert term in (2.4) dominates. Then as will be shown later, the singularity is moderated and if we include usual matter, the singularity will not appear eventually. Since $w+1 = \frac{2(n+2)}{3(n+1)(2n+1)}$, it follows $w < -1$ when $-1 < n < -\frac{1}{2}$ (the effective phantom phase for fractional positive power of the curvature). Here it is assumed $n > -1$, so that the Einstein term dominates in (2.5) when the curvature is small. We should also note when $n > 0$, as $\frac{2(n+2)}{3(n+1)(2n+1)} > 0$, w is always greater than -1. Then the theory with negative power of the curvature like $\frac{1}{R}$-gravity does not produce effective phantom although it may produce effective quintessence.

The results (2.7) are valid when the curvature is small but near the Big Rip singularity, the curvature becomes large and (2.7) is not valid. The qualitative behavior when the curvature is large can be found from the potential.

The qualitative behavior of the potential when $-1 < n < -\frac{1}{2}$ and $\gamma > 0$ is given in Figure 1a). In order that σ is real, however, Eq.(2.5) tells $R \sim A > (-n\gamma)^{\frac{1}{n+1}}$. Then the curvature cannot be small and the expressions (2.7) are not valid. When $A < (-n\gamma)^{\frac{1}{n+1}}$, instead of (2.5), one may define $\sigma = -\ln\left(-1 - n\gamma A^{-n-1}\right)$. As shown in [5], however, the anti-gravity appears in this case and instead of (2.2), we obtain $S_E = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\left(-R + \frac{3}{2}g^{\rho\sigma}\partial_\rho\sigma\partial_\sigma\sigma - \tilde V(\sigma)\right)$ with potential $\tilde V(\sigma) = -\mathrm{e}^{\sigma} g\left(\mathrm{e}^{-\sigma}\right) - \mathrm{e}^{2\sigma} f\left(g\left(\mathrm{e}^{-\sigma}\right)\right)$. Hence, when $-1 < n < -\frac{1}{2}$ and $\gamma > 0$, the region $A < (-n\gamma)^{\frac{1}{n+1}}$ is not physical. Then we should assume $\gamma < 0$. In case $-1 < n < -\frac{1}{2}$ and $\gamma < 0$, there does not appear the extremum in the potential if curvature is positive. The qualitative behavior of the potential is given in Figure 1b). The Eq.(2.6) shows that the potential is negative and is unbounded below when A is large.

In order to consider the region where the curvature is large for the case $-1 < n < -\frac{1}{2}$ and $\gamma < 0$, the following potential is taken

$$V = -V_0\left(-\sigma\right)^{\alpha}\ , \quad V_0 \equiv -\frac{n+1}{n}\left(n\gamma\right)^{\frac{1}{n+1}} > 0\ , \quad \alpha \equiv \frac{n}{n+1} < -1\ . \qquad (2.8)$$

The σ-equation of motion and the FRW equation in the Einstein frame are $0 = -3\left(\frac{d^2\sigma}{dt_E^2} + 3H\frac{d\sigma}{dt_E}\right) - V'(\sigma)$ and $\frac{6}{\kappa^2}H_E^2 = \frac{3}{2}\left(\frac{d\sigma}{dt_E}\right)^2 + V(\sigma)$. A consistent solution is

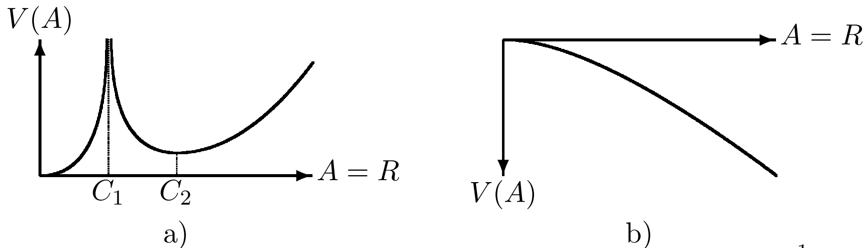

Figure 1: a) A qualitative behavior of the potential with $-1<n<-\frac{1}{2}$ and $\gamma>0$. Here $C_1\equiv(-n\gamma)^{\frac{1}{n+1}}$ and $C_2\equiv(-(n+2)\gamma)^{\frac{1}{n+1}}$ In the region with $A<(-n\gamma)^{\frac{1}{n+1}}$, there appears the anti-gravity phase. b). A qualitative behavior of the potential with $-1<n<-\frac{1}{2}$ and $\gamma<0$. The potential is monotonically decreasing.

given by

$$H_E=0,\ \sigma=-\sigma_0\left(t_{sE}-t_E\right)^{\beta},\ \sigma_0\equiv\left(\frac{2V_0}{3\beta^2}\right)^{\frac{1}{2-\alpha}},\ \beta\equiv\frac{2}{2-\alpha}=\frac{2(n+1)}{n+2}\ . \tag{2.9}$$

Here t_{sE} is a constant of integration. Since $-1<n<-\frac{1}{2}$, it follows $0<\beta<\frac{2}{3}$. The spacetime in the Einstein frame is flat and the scale factor $a_E=a_0$ is a constant which is not the case in the physical Jordan frame, where they are finite even at $t_E=t_{sE}$ although there is a cut singularity there. The cut singularity, however, makes the physical curvature divergent. In fact, when $t_E\sim t_{sE}$, t is given by $t=t_s+t_E-t_{sE}$ with a constant of the integration t_s. In the Jordan frame, the scale factor a is given by $a(t)\sim a_0\mathrm{e}^{-\frac{\sigma_0}{2}(t_s-t)^{\beta}}$ and the Hubble parameter $H\equiv\frac{1}{a}\frac{da}{dt}$ behaves as $H=\frac{1}{2}\frac{d\sigma}{dt}=\frac{1}{2}\frac{d\sigma}{dt_E}\frac{dt_E}{dt}=\frac{\beta\sigma_0}{2}\left(t_{sE}-t_E\right)^{\beta-1}\mathrm{e}^{-\frac{\sigma}{2}}\sim\frac{\beta\sigma_0}{2}\left(t_s-t\right)^{\beta-1}$. Since $-1<\beta-1<-\frac{1}{3}<0$, H diverges at $t=t_s$ as well as the scalar curvature $R=6\frac{dH}{dt}+12H^2$. In case of the Big Rip singularity, H behaves as $H\sim\frac{h_0}{t_s-t}$. Hence, the behavior of H is moderated. In case of the Barrow model [15] (see also [16, 17]), H behaves as $H\sim h_0+h_1\left(t_s-t\right)^{\alpha}$ with constant h_0 and h_1. As $0<\alpha<1$, the behavior of H here is more singular than that in [15]. The singularity in the present case is moderated and a does not diverge. With the usual matter inclusion such a singularity will not appear. It is interesting to note that for the model [15], the singularity is moderated (or even disappears) if the quantum corrections are taken into account [18]. Note also that usual, infinite-time singularity is still possible even in consistent modified gravity (for earlier, related discussion of infinite time, future singularity in $f(R)$ gravity, see [19]).

In accord with the proposal of ref.[4] one may add a term proportional to R^2 to the action (2.4)

$$S=\frac{1}{\kappa^2}\int d^4x\sqrt{-g}\left(R-\gamma R^{-n}+\eta R^2\right)\ . \tag{2.10}$$

In this situation the unified theory permits early time inflation as well as late time acceleration and does not contain the number of instabilities. The function $f(A)$ (2.5) is modified to be $f(A)=A-\gamma A^{-n}+\eta A^2$. We now assume $-1<n<-\frac{1}{2}$, $\gamma<0$, and

$\eta > 0$. Then $\sigma = -\ln\left(1 + n\gamma A^{-n-1} + 2\eta A\right)$. Although $1+n\gamma A^{-n-1}+2\eta A > 0$, since $\frac{d\sigma}{dA} = -\frac{f''(A)}{f'(A)} = -\frac{-n(n+1)\gamma A^{-n-2}+2\eta}{1+n\gamma A^{-n-1}+2\eta A}$, there is a branch point, where $\frac{d\sigma}{dA} = 0$ or $f''(A) = 0$, at $A = A_0 \equiv \left\{\frac{n(n+1)\gamma}{2\eta}\right\}^{\frac{1}{n+2}}$ or $\sigma = \sigma_0 \equiv -\ln\left(1 + (n+2)\left(\frac{2\eta}{n+1}\right)^{\frac{n+1}{n+2}}(n\gamma)^{\frac{1}{n+2}}\right)$. The potential $V(A)$ has the following form:

$$V(A) = \frac{(n+1)\gamma A^{-n} + \eta A^2}{\left(1 + n\gamma A^{-n-1} + 2\eta A\right)^2} \ . \tag{2.11}$$

The behavior when A is small is not changed from the case in (2.4). On the other hand, when A is large, $V(A)$ goes to a constant: $V(A) \to \frac{1}{4\eta}$. The potential $V(A)$ vanishes at $A = A_1 \equiv \left\{\frac{-(n+1)\gamma}{\eta}\right\}^{\frac{1}{n+2}}$. Note $A_0 < A_1$ since $0 < \frac{n\gamma}{2} < -\gamma$. $V'(A)$ has an extremum at $A = A_0$ since $V'(A) = \frac{\left\{-n(n+1)\gamma A^{-n-2}+2\eta\right\}A\left\{1-(n+2)\gamma A^{-n-1}\right\}}{(1+n\gamma A^{-n-1}+2\eta A)^3}$. Since there is a branch point at $\sigma = \sigma_0$,if we start from the small curvature, the growth of the curvature stops at $R = A_0$, where $\sigma = \sigma_0$. In fact, at the branch point, where $f''(A) = 0$, the mass m_σ of σ becomes infinite since $m_\sigma \propto \frac{d^2V}{d\sigma^2} = \frac{f'(A)}{f''(A)}\frac{d}{dA}\left(\frac{f'(A)}{f''(A)}\frac{dV(A)}{dA}\right) = -\frac{3}{f''(A)} + \frac{A}{f'(A)} + \frac{2f(A)}{f'(A)^2} \to +\infty$. Note also $f''(0) < 0$ when $A < A_0$. Then the growth of σ is finished at $\sigma = \sigma_0$. Hence, adding R^2 term, there does not occur cosmic doomsday but the universe ends up in deSitter phase. The scale factor a is given by $a(t) \sim a_0 \mathrm{e}^{t\sqrt{\frac{A_0}{12}}}$ with a constant a_0. Note that the quantities in the Einstein frame are different from those in the Jordan frame by almost constant factor as $a_E \sim \mathrm{e}^{\frac{\sigma_0}{2}}$ or $dt_E \sim \mathrm{e}^{\frac{\sigma_0}{2}}dt$. This supports our point of view that cosmic doomsday in such theory does not occur because after the late time acceleration the universe starts new inflationary era. It is also important to stress that as the mass of σ becomes very large, there is no problem about the equivalence principle since σ cannot mediate the force. In other words, unlike to BD theory, such a model may pass the solar system test provided by VLBI experiment.

3. Modified gravity coupled with matter

It is very interesting that modified gravity which can be made consistent one [4] may help in the resolution of dark energy problem in various ways as it suggests gravitational alternative for dark energy. In particular, as we will show below it may give phantom dark energy without necessity to introduce the (negative kinetic energy) phantom scalar theory. In fact, the matter is taken to be the usual ideal fluid.

We now consider the system of the modified gravity coupled with matter:

$$S = \int d^4x\sqrt{-g}\left\{f(R) + L_m\right\} \ . \tag{3.1}$$

Here $f(R)$ is an adequate function of the scalar curvature and L_m is a matter Lagrangian. Then the equation of the motion is given by $0 = \frac{1}{2}g_{\mu\nu}f(R) - R_{\mu\nu}f'(R) - \nabla_\mu\nabla_\nu f'(R) - g_{\mu\nu}\nabla^2 f'(R) + \frac{1}{2}T_{\mu\nu}$. Again, the FRW spacetime is considered. The ideal fluid is taken as the matter with the constant w: $p = w\rho$. Then from the energy conservation law it follows $\rho = \rho_0 a^{-3(1+w)}$. In a some limit, strong cuvature or weak one, $f(R)$ may behave as $f(R) \sim f_0 R^\alpha$, with constant f_0 and α. An exact solution of the equation of motion is found to be

$$a = a_0 t^{h_0} \ , \quad h_0 \equiv \frac{2\alpha}{3(1+w)} \ ,$$

$$a_0 \equiv \left[-\frac{6f_0 h_0}{\rho_0} \left(-6h_0 + 12h_0^2\right)^{\alpha-1} \left\{(1-2\alpha)(1-\alpha) - (2-\alpha)h_0\right\} \right]^{-\frac{1}{3(1+w)}} . \tag{3.2}$$

When $\alpha = 1$, the result $h_0 = \frac{2}{3(1+w)}$ in the Einstein gravity is reproduced. Note that stability issue should be carefully investigated here. However, even if the solution is instable the decay time could be very big due to the fact that coupling constant of modified gravity term is very small. From another side, when finite-time future singularity occurs it may be resolved by the account of quantum effects [18, 5].

The effective $w_{\rm eff}$ may be defined by $h_0 = \frac{2}{3(1+w_{\rm eff})}$. By using (3.2), one finds

$$w_{\rm eff} = -1 + \frac{1+w}{\alpha} \ . \tag{3.3}$$

Hence, if w is greater than -1 (effective quintessence or even usual ideal fluid with positive w), when α is negative, we obtain the effective phantom phase where $w_{\rm eff}$ is less than -1. This is different from the case of pure modified gravity. Moreover, when $\alpha > w+1$ (it can be even positive), $w_{\rm eff}$ could be negative (for negative w). Hence, it follows that modified gravity minimally coupled with usual (or quintessence) matter may reproduce quintessence (or phantom) evolution phase for dark energy universe in an easier way than without such coupling.

One may now take $f(R)$ as in (2.10). When the cuvature is small, the second term becomes dominant and one may identify $f_0 = -\frac{\gamma}{\kappa^2}$ and $\alpha = -n$. Then from (3.3), it follows $w_{\rm eff} = -1 - \frac{1+w}{n}$. Hence, if $n > 0$, we have an effective phantom even if $w > -1$. Usually the phantom generates the Big Rip singularity. However, near the Big Rip singularity, the curvature becomes large and the last term becomes dominant. In this case $\alpha = 2$ and $w_{\rm eff} = \frac{-1+w}{2}$. Then if $w > -1$, it follows $w_{\rm eff} > -1$, which prevents the Big Rip singularity (makes phantom phase transient) as is described in the previous section. To conclude, it looks quite promising that modest modification of General Relativity coupled to ideal fluid matter leads to effective dark energy universe in the very natural way.

4. Discussion

In summary, the gravity theory with negative (like $1/R$) and positive (quadratic)

powers of scalar curvature shows the number of features which are desireable to explain the accelerating dark energy universe:
1. It passes solar system tests (VLBI experiment). Thanks to higher derivative R^2 term, the gravitationally bound objects (like Sun or Earth) are stable as well as in GR. Of course, some fine-tuning of coupling constants is necessary as is shown in [4]. (It was mentioned already in ref.[1] that the coefficient of $1/R$-term should be extremely small.)
2. Newtonian limit is recovered just at the above values for coupling constants.
3. The gravitational dark energy dominance is explained simply by the universe expansion. Moreover, when modified gravity is coupled with usual matter it is easier (less deviations from GR) to get the effective (phantom or quintessence) dark energy universe regime.
4. The presence of R^2 term in consistent modified gravity prevents the development of the cosmic doomsday.

Thus, the consistent modified gravity remains to be viable candidate for the explanation of dark energy as the gravitational phenomenon. Nevertheless, only future, more precise astrophysical/gravitational data will prove if it is the time for new gravitational physics to enter the game.

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